Longitudinal Waves

28TH SEPTEMBER 2020

Sound wave in gases

- In deriving the wave equation for the longitudinal wave, $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$, let consider a **fixed mass of gas in a volume**.

- Under the influence of the longitudinal or sound wave

the pressure	P_0	becomes	$P_0 + p$
the volume	V_0	becomes	$V_0 + v$
the density	ρ_0	becomes	$\rho_0 + \rho_d$

Disturbed state parameters

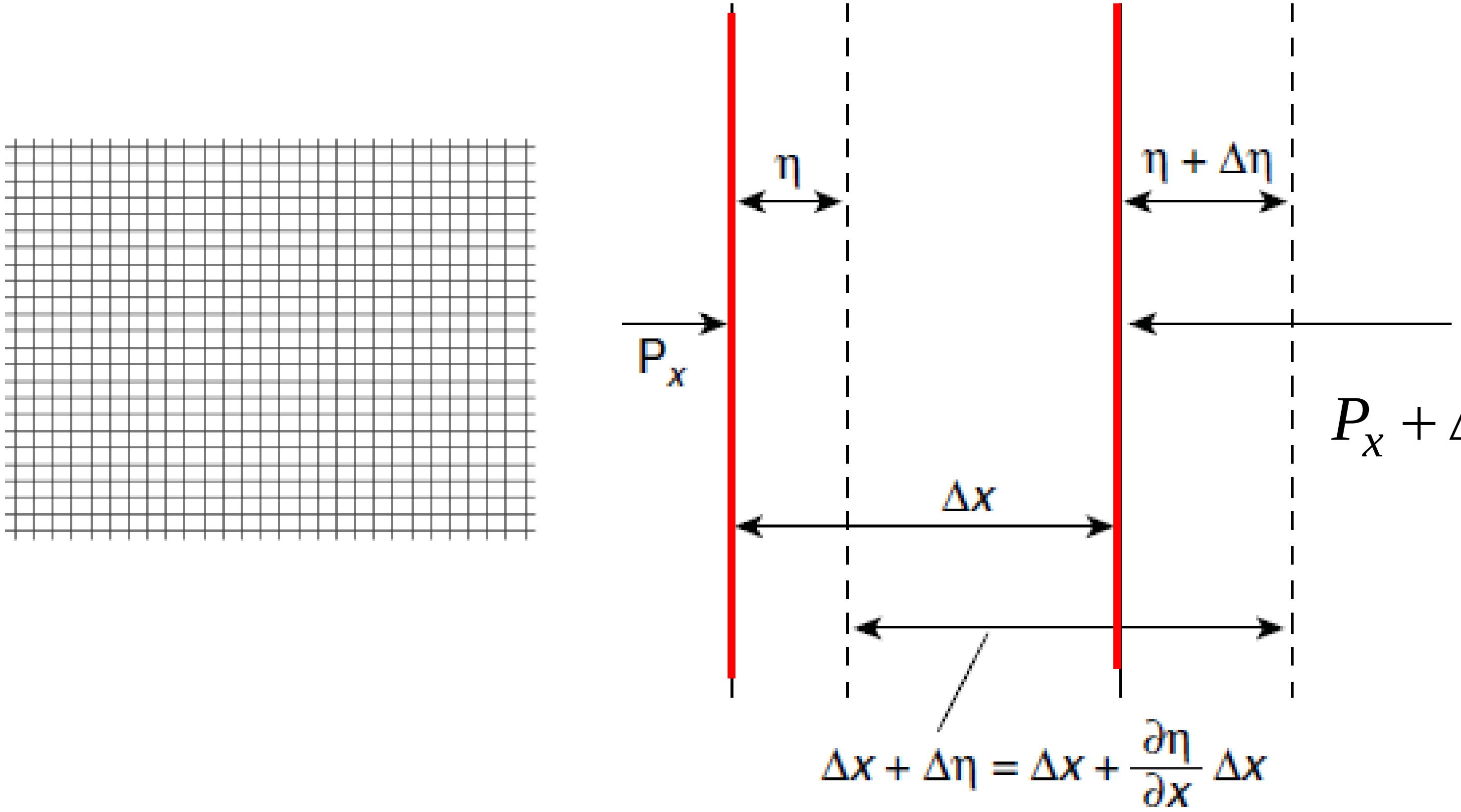
Equilibrium state parameters

Typical changes in the medium due to sound waves

- The fractional volume (***dilatation***): $\delta = v / V_0 \approx 10^{-3}$

- The fractional change of density (***condensation***): $s = \rho_d / \rho_0 \approx 10^{-3}$

- The maximum pressure amplitude for ordinary sound wave : $p_m = 2 \times 10^{-5}\ \text{Pa}$
- For **a fixed mass of gas**, $\rho_0 V_0 = \rho V = \rho_0 V_0 (1+\delta)(1+s)$
- This gives $s = -\delta$ to a very close approximation.

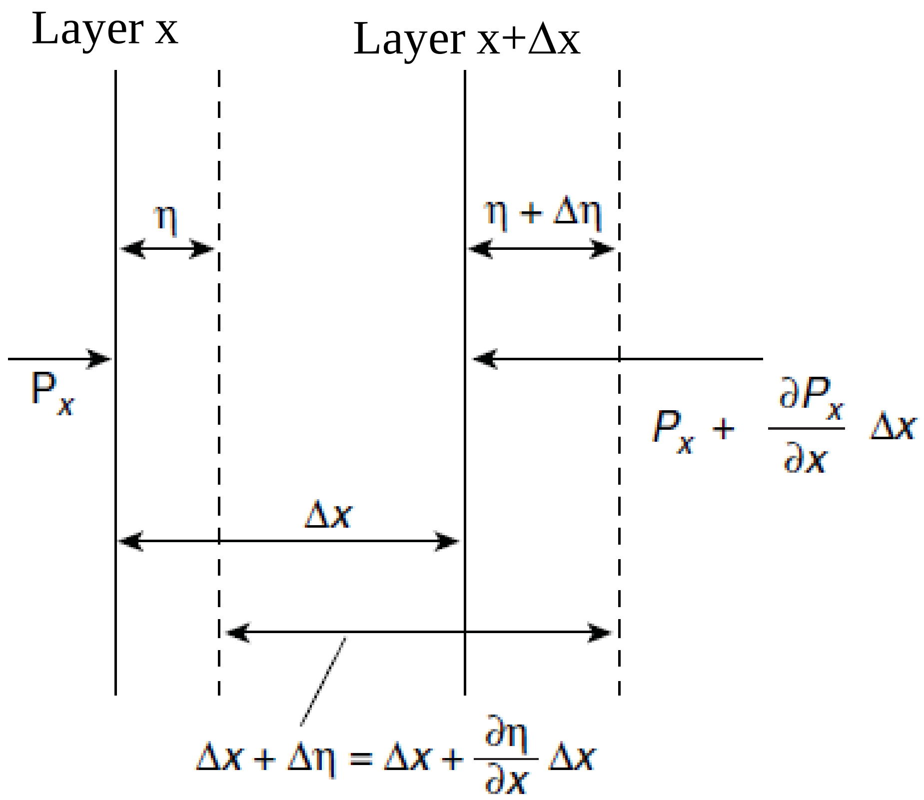


$$P_x + \Delta P_x = P_x + \frac{\partial P_x}{\partial x}\Delta x$$

This factor indicates how pressure is changed as the distance changes

Figure 6.1 Thin element of gas of unit cross-section and thickness Δx displaced an amount η and expanded by an amount $(\delta\eta/\partial x)\Delta x$ under the influence of a pressure differene $-(\partial P_x/\partial x)\Delta x$

The wave equation of sound wave in gas



Thin element of gas of **unit cross-section** and thickness Δx displaced and amount η under the influence of a pressure difference.

- When there is no disturbance, the volume of the medium under interest is $V = A\Delta x$. **For a unit cross section, $V = \Delta x$**.

When there is a sound wave disturbance at time t, the particles in the layer x are displaced a distance $\eta_1 = \eta(x, t)$

And the particles in the layer x + Δx are displaced a distance

$$\eta_2 = \eta(x+\Delta x, t)$$

- Three possible changes of volume may be observed :

(1) $\eta_1 = \eta_2$: a **constant** volume shifted to the right

(2) $\eta_1 < \eta_2$: an **increased** volume shifted to the right

(3) $\eta_1 > \eta_2$: a **decreased** volume shifted to the right

- The increase of the element thickness Δη can be written as

$$\Delta\eta = \eta_2 - \eta_1 = \eta(x+\Delta x, t) - \eta(x, \mathrm{t})$$

- This can be written in the derivative form meaning “the change of η (displacement) with respect to the change of x (distance)” as

$$\frac{\partial\eta}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta\eta}{\Delta x} = \frac{\eta(x+\Delta x, t) - \eta(x, \mathrm{t})}{\Delta x}$$

- Therefore, the increase in the thickness Δx of the element of unit cross section is

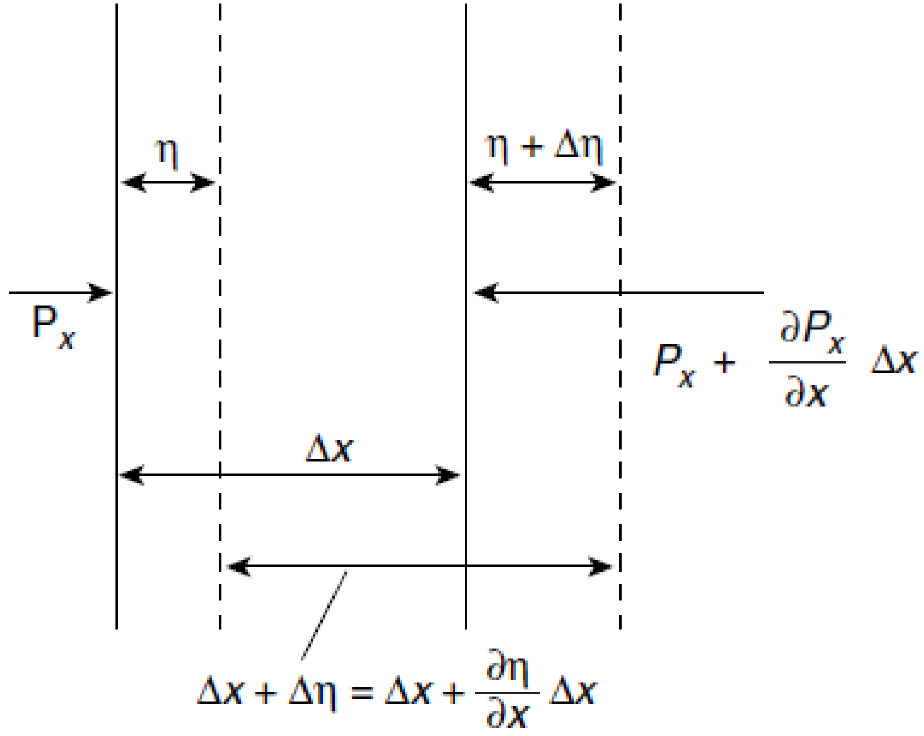
$$\Delta\eta = \frac{\partial\eta}{\partial x}\Delta x$$

- And $\delta = \dfrac{v}{V_0} = \dfrac{(\text{unit area})\left(\dfrac{\partial\eta}{\partial x}\right)\Delta x}{(\text{unit area})\Delta x} = \left(\dfrac{\partial\eta}{\partial x}\right) = -s$

Longitudinal strain

The wave equation of sound wave in gas (contd.)

The medium is **deformed** because the pressures along the x-axis on either side of the thin element are not in balance. The **net force acting** on the gas element is given



$$P_x - P_{x+\Delta x} = \left[P_x - \left(P_x + \frac{\partial P_x}{\partial x}\Delta x\right)\right]$$

$$= -\frac{\partial P_x}{\partial x}\Delta x = -\frac{\partial}{\partial x}\left(P_0 + p\right)\Delta x$$

$$= -\frac{\partial p}{\partial x}\Delta x$$

Note : (1) *p* is the excess pressure or pressure change.

(2) the unit area cross section is used in this study

The wave equation of sound wave in gas (contd.)

According to the Newton's 2nd Law, $$-\frac{\partial p}{\partial x}\Delta x = (\rho_0 \Delta x)\frac{\partial^2 \eta}{\partial t^2}$$

- The **pressure change *p*** can be written in terms of displacement η when using the bulk modulus B_a (=-VdP/dV) under the condition of the **adiabatic process** (Laplace's idea).
- $\therefore p = -B_a(dV/V) = -B_a(v/V_0) = -B_a\delta = -B_a(\partial\eta/\partial x)$

From slide No. 6

- This gives

$$B_a\frac{\partial^2 \eta}{\partial x^2} = \rho_0\frac{\partial^2 \eta}{\partial t^2}$$

- This can be expressed as a final form of the wave equation as

$$\frac{\partial^2 \eta}{\partial x^2} = \frac{\rho_0}{\gamma P}\frac{\partial^2 \eta}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 \eta}{\partial t^2}$$

Where $B_a = \gamma P$ representing the elastic property of the gas and c is the speed of sound in gas.

What if the sound medium obeys the isothermal process? (Newton's idea)

- For the isothermal thermal process : PV = constant.
- According to the definition of the bulk modulus : $B = -V \, {}^{dP}\!/_{dV}$.
- Therefore, $B_{iso} = P$.
- This leads the equation of motion of the sound wave in a gas medium to be

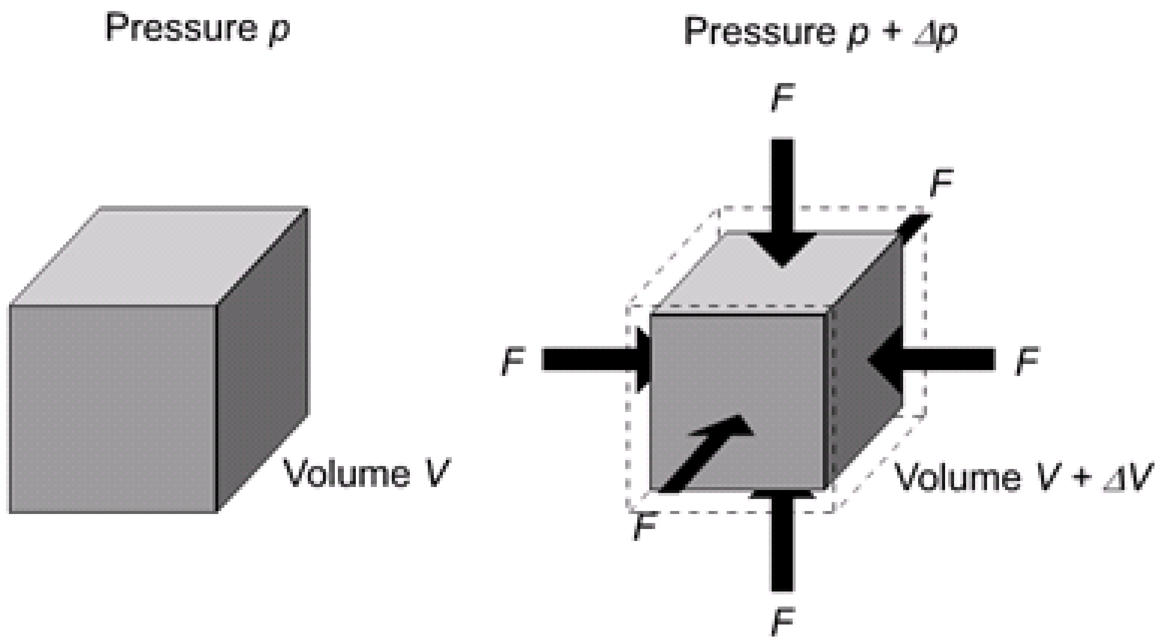
$$B_{iso} \frac{\partial^2 \eta}{\partial x^2} = \rho_0 \frac{\partial^2 \eta}{\partial t^2}$$

$$\frac{\partial^2 \eta}{\partial x^2} = \frac{\rho_0}{P} \frac{\partial^2 \eta}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 \eta}{\partial t^2}$$

- The sound speed can be found from $c = \sqrt{\frac{P}{\rho_0}} = \sqrt{\frac{RT}{M}} = \sqrt{\frac{kT}{m_0}}$

M =molecular weight
k = R/N
N = Avogadro's number
m0 = mass of an individual molecule

Bulk modulus



Pressure increases,
Volume decreases

- The Bulk Modulus tells us just how compressible materials are.

$$Bulk\ Modulus\ K = \frac{Bulk\ Stress}{Bulk\ Strain} = \frac{\left(\frac{Force}{Area}\right)}{\left(\frac{\Delta V}{V}\right)} = -\frac{\Delta p}{\left(\frac{\Delta V}{V}\right)}$$

This minus sign makes a positive bulk modulus.

Example 1 : Bulk modulus of water

Compute the bulk modulus of water from the following data:

- **Initial volume = 100.5 l., Final volume = 100 l., Pressure increase 100.0 atm (1 atm = 1.013 x 10^5 Pa)**

TABLE 9–1 Elastic Moduli

Material	Young's Modulus, E (N/m^2)	Shear Modulus, G (N/m^2)	Bulk Modulus, B (N/m^2)
Solids			
Iron, cast	100×10^9	40×10^9	90×10^9
Steel	200×10^9	80×10^9	140×10^9
Brass	100×10^9	35×10^9	80×10^9
Aluminum	70×10^9	25×10^9	70×10^9
Concrete	20×10^9		
Brick	14×10^9		
Marble	50×10^9		70×10^9
Granite	45×10^9		45×10^9
Wood (pine) (parallel to grain)	10×10^9		
(perpendicular to grain)	1×10^9		
Nylon	$\approx 3 \times 10^9$		
Bone (limb)	15×10^9	80×10^9	
Liquids			
Water			2.0×10^9
Alcohol (ethyl)			1.0×10^9
Mercury			2.5×10^9
Gases[†]			
Air, H_2, He, CO_2			1.01×10^5

[†]At normal atmospheric pressure; no variation in temperature during process.

https://iitsat.com/compute-the-bulk-modulus-of-water-from-the-following-data-initial-volume-100-0-litre-pressure-increase-100-0-atm-1-atm-1-013-x105-pa-final-volume-100-5-l/

Example 2 : What is the speed of sound wave at sea level?

- Recall the speed of sound wave : $c = \sqrt{\dfrac{\gamma P}{\rho_0}}$

- For air sea level, the pressure is ……………. and the density of air is…………
- And $\gamma = 1.4$.   This gives c =………………
- The speed of sound wave is proportional to the pressure P and inversely proportional to density ρ_0 .

Calculate the speed of sound wave using the Newtown's formula!

Practical formula for speed of sound wave in dry air as a function of temperature

- According to

$$c = \sqrt{\frac{\gamma P}{\rho_0}} = \sqrt{\frac{\gamma nRT}{\rho_0 V}} = \sqrt{\frac{\gamma mRT}{\rho_0 MV}} = \sqrt{\frac{\gamma RT}{M}}$$

- Substitute

$$\mathrm{R} = 8.314 \ \ \mathrm{J/mol \cdot K}$$

$$\mathrm{M\ (molar\ mass\ of\ dry\ air)} = 0.0289 \ \ \mathrm{kg/mol}$$

$$\mathrm{T} = 273.15 + \mathrm{t}\left({}^{o}c\right)$$

- The speed of sound wave in dry air can be written as

Phase relationships

- For a travelling wave propagating in the **positive x-direction**

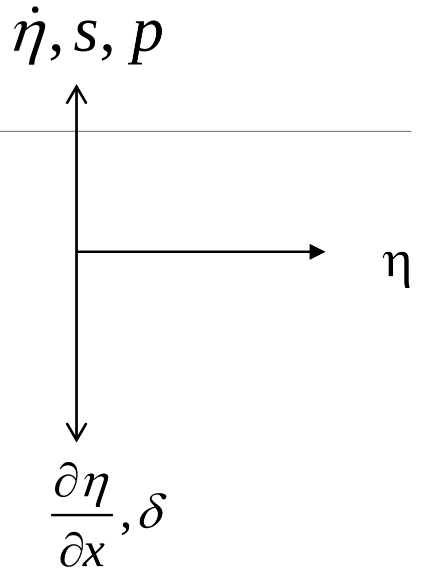
$$\eta = \eta_m e^{i(\omega t - kx)}; \frac{d\eta}{dt} = \dot{\eta} = i\omega\eta$$

From slide No. 6

$$\delta = \frac{\partial \eta}{\partial x} = -ik\eta = -s$$

From slide No. 8

$$p = iB_a k\eta$$

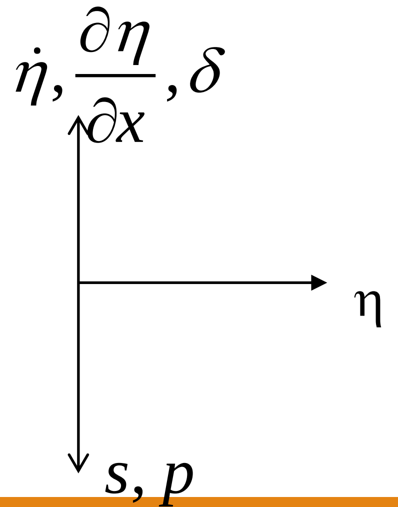


- For a travelling wave propagating in the **negative x-direction**

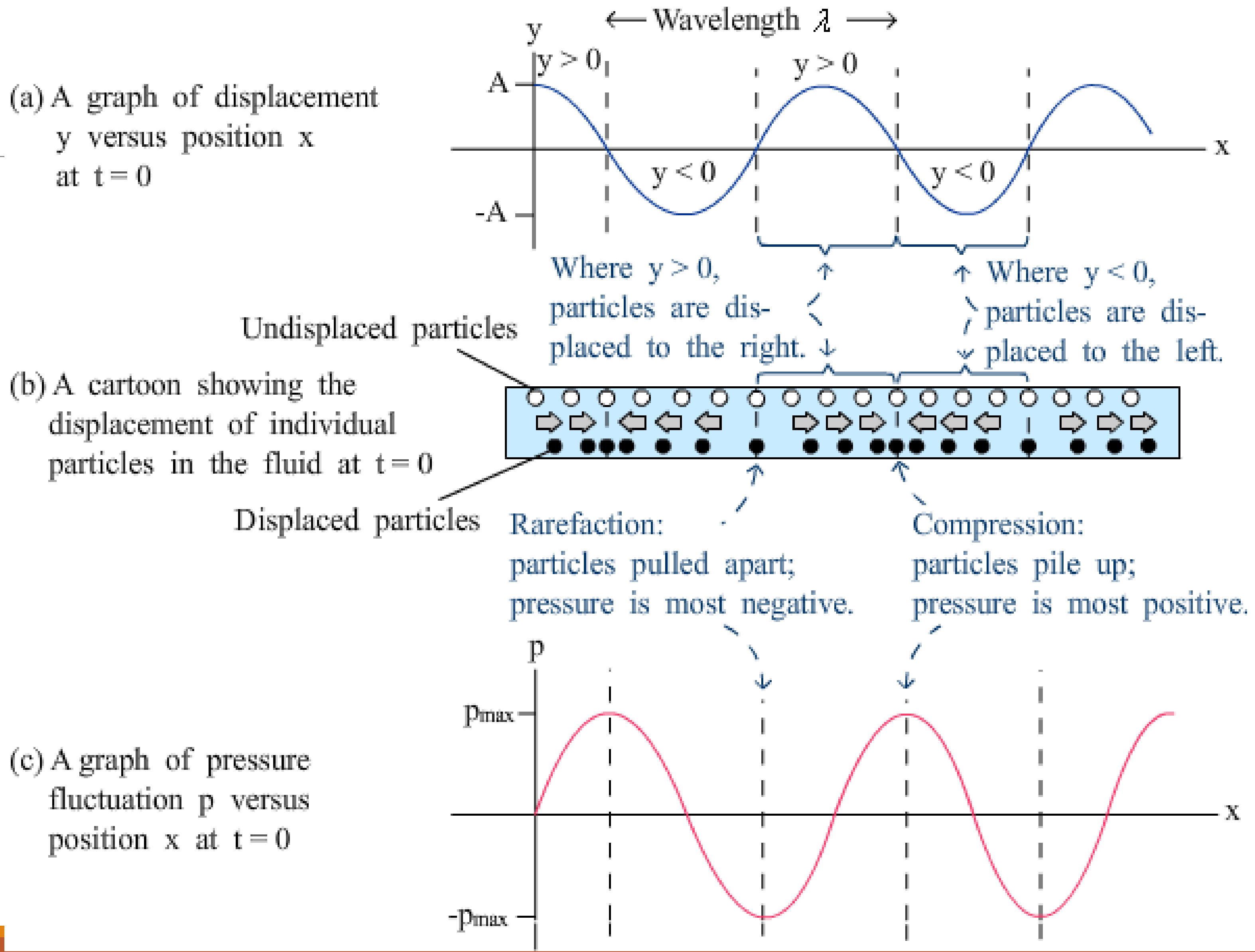
$$\eta = \eta_m e^{i(\omega t + kx)}; \dot{\eta} = i\omega\eta$$

$$\delta = ik\eta = -s$$

$$p = -iB_a k\eta$$



Pressure and displacement in air column



Notice the phase difference between the displacement and pressure

They are $^{\pi}/_{2}$ difference in phase.

http://ap.polyu.edu.hk/apahthua/College%20Physics/ch16_1.html

Energy distribution in travelling sound waves

Determine the average values of the kinetic and potential energy density in the sound wave.

- Each element has a kinetic energy per unit cross section given by $\Delta E_{kin} = \frac{1}{2}\rho_0 \Delta x \dot{\eta}^2$
- With a displacement

$$\eta = \eta_m \cos(\omega t - kx)$$
$$\therefore \dot{\eta} = (-\omega \eta_m) \sin(\omega t - kx) = \dot{\eta}_m \sin(\omega t - kx)$$

- The **space average kinetic energy density** (kinetic energy per volume) is written as

$$\overline{\Delta E}_{kin} = \frac{1}{2}\rho_0 \overline{\dot{\eta}^2}$$

- .
- This gives

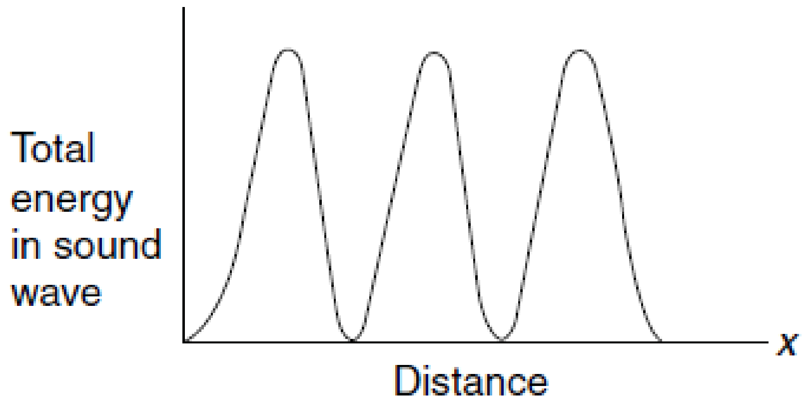
$$\overline{\Delta E}_{kin} = \frac{1}{4}\rho_0 \dot{\eta}_m^2 = \frac{1}{4}\rho_0 \omega^2 \eta_m^2$$

Energy distribution in travelling sound waves

- The average potential energy density (potential energy per volume) is written in a similar for as

$$\overline{\Delta E}_{pot} = \frac{1}{4}\rho_0 \dot{\eta}_m^2$$

- In fact, the total energy content of sound wave in an element Δx is given as
- $$\Delta E_{total} = \Delta E_{kin} + \Delta E_{pot} = \frac{1}{4}\rho_0 \dot{\eta}^2\, \Delta x + \frac{1}{4}\rho_0 \dot{\eta}^2\, \Delta x$$
- This indicates that the element possess maximum (or minimum) potential and kinetic energy at the same time.



At points of no velocity, there is no compression and the particles do not possess energy at these points.

At points of maximum velocity, there is compression and the particles possess maximum energy.

Intensity of sound waves

- Intensity is the measure of the energy flux.

power

Energy flux =the rate of energy crossing unit area

= total energy density × wave velocity c

- Therefore, the intensity is written as

$$I = \boxed{\frac{1}{2}\rho_0\dot{\eta}_m^2} \times c = \frac{1}{2}(c\rho_0)\omega^2\eta_m^2;\ \because\ \dot{\eta}_m = -\omega\eta_m$$

From slide No. 16

- Dynamic range of the audible sound intensity (**Sound intensity level**) is between 0 dB to 120 dB.
- The lowest and the highest intensity levels are based on normal sound waves range in intensity between 10^{-12} W/m^2 and 1 W/m^2

Example 3 : sound intensity level

A point source of sound emits 50,000 joules of sound energy every 20 seconds. At a distance 100 meters from the source, what is the intensity of the sound (in decibels), if no energy is lost in the intervening space?

Example 4 : displacement amplitude

Barely audible sound in air has an intensity of 10^{-12} w/m^2.

Determine the displacement amplitude of an air molecule for sound at this level at 500 Hz in an ambient temperature of 25^0C.

Specific Acoustic Impedance

Specific Acoustic Impedance = excess pressure/particle velocity = $p/\dot{\eta}$

- For a wave in a positive x-direction:

$$p = B_a s = iB_a k\eta \quad \text{and} \quad \dot{\eta} = i\omega\eta \quad \text{where} \quad \eta = \eta_m e^{i(\omega t - kx)}$$

$$\text{so that} \quad \frac{p}{\dot{\eta}} = \frac{B_a k}{\omega} = \frac{B_a}{c} = \boxed{\rho_0 c} \quad \text{where} \quad \frac{B_a}{\rho_0} = \frac{\gamma P}{\rho_0} = c^2$$

Referring to slide No. 14

- For a wave in a negative x-direction : $\boxed{-\rho_0 c}$
- The unit is given as kg m^{-2} s^{-1}.
- Specific acoustic impedances of air, water and steel in the unit are 400, 1.45 x 10^6 and 3.9 x 10^7 kg m^{-2} s^{-1}, respectively.

The importance of the specific acoustic impedance

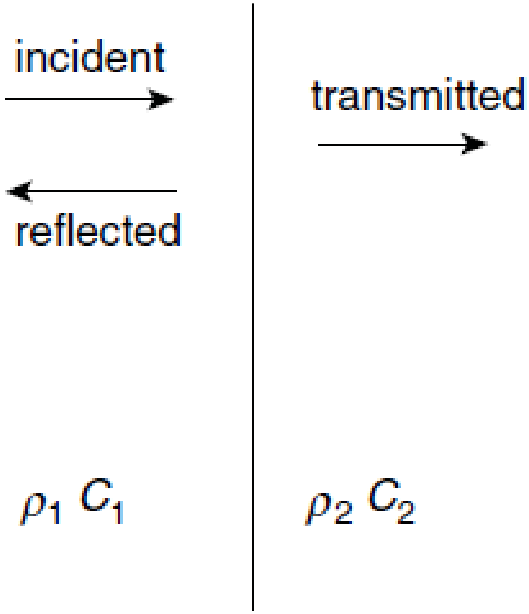
- **The determination of acoustic transmission and reflection at the boundary of two materials having different specific acoustic impedances**.
- When a sound wave meets a boundary between two media of different specific acoustic impedances, two boundary conditions must be met.

    (1) the continuity of particle velocity

    (2) the continuity of the acoustic excess pressure p

- Physically, this ensures that the two media are in complete contact everywhere across the boundary.

Reflection and transmission coefficients



- According to the boundary conditions

$$\dot{\eta}_i + \dot{\eta}_r = \dot{\eta}_t$$

$$p_i + p_r = p_t$$

- Because $p = \rho c \dot{\eta}$ (**from slide #21**) and substituting corresponding excess pressures into above equations and determine both coefficients.

Reflection and transmission coefficients of particle velocity

$$\frac{\dot{\eta}_r}{\dot{\eta}_i} = \frac{\eta_r}{\eta_i} = \frac{Z_1 - Z_2}{Z_1 + Z_2},$$

$$\frac{\dot{\eta}_t}{\dot{\eta}_i} = \frac{\eta_t}{\eta_i} = \frac{2Z_1}{Z_1 + Z_2}$$

Reflection and transmission coefficients of pressure

$$\frac{p_r}{p_i} = \frac{Z_2 - Z_1}{Z_1 + Z_2} = -\frac{\dot{\eta}_r}{\dot{\eta}_i},$$

$$\frac{p_t}{p_i} = \frac{Z_2 \dot{\eta}_t}{Z_1 \dot{\eta}_i} = \frac{2Z_2}{Z_1 + Z_2}$$

Example 5 : boundary conditions

Standing acoustic waves are formed in a tube of length l with (a) both ends open and (b) one end open and the other closed. If the particle displacement

$$\eta = (A\cos kx + B\sin kx)\sin\omega t$$

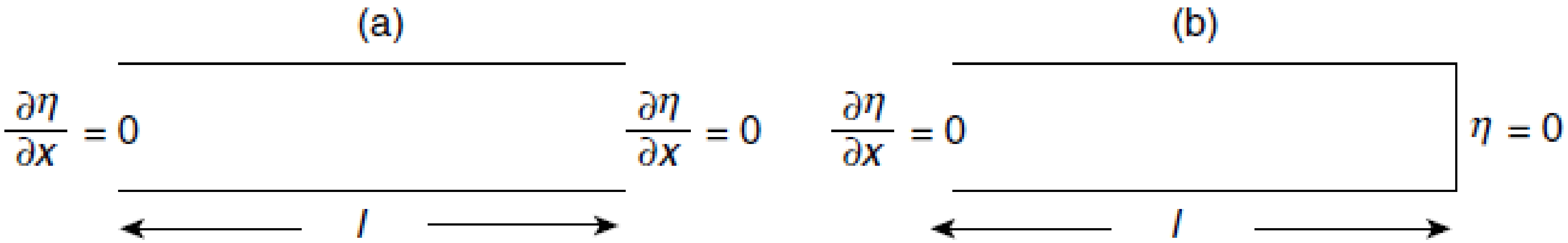
and the boundary conditions are as shown in the diagrams, show that for

$$\text{(a)} \quad \eta = A\cos kx \sin\omega t \quad \text{with} \quad \lambda = 2l/n$$

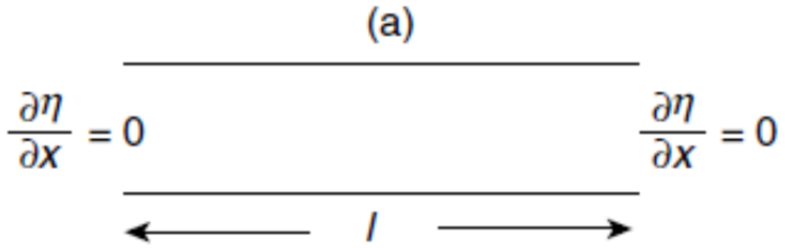
and for

$$\text{(b)} \quad \eta = A\cos kx \sin\omega t \quad \text{with} \quad \lambda = 4l/(2n+1)$$

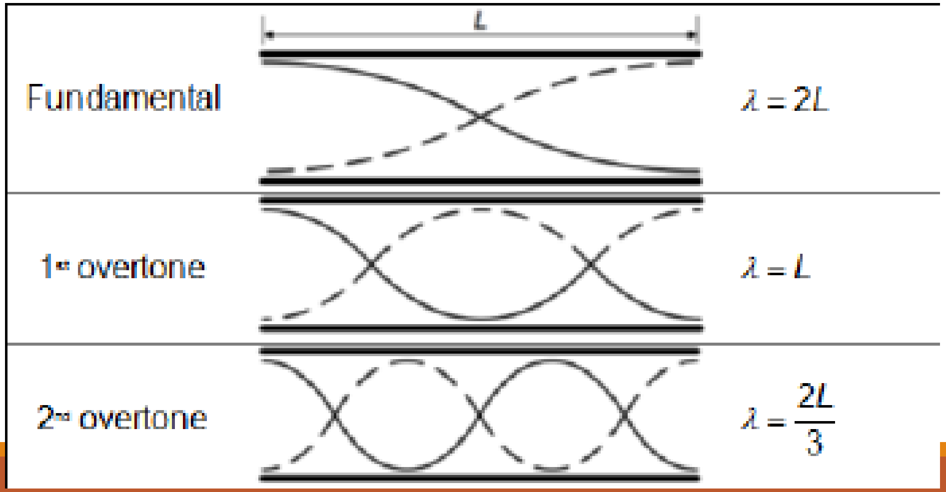
Sketch the first three harmonics for each case.



Solution



- Consider case (a)
- Expression of the standing wave is given as $\eta = (A \cos kx + B \sin kx) \sin \omega t$
- Applying the boundary condition on the left hand side :$\frac{\partial \eta}{\partial x} = 0$, we found that B =0.
- Therefore, $\eta = A \cos kx \sin \omega t$.
- Applying the boundary condition on the right hand side : $\frac{\partial \eta}{\partial x} = 0$, we found that $\sin kx = n\pi$.
- Therefore, $\lambda = \frac{2l}{n}$; where n = 1, 2, 3,…
- Standing wave for the first three harmonics :



Your turn to try case (b)!

Reflection and transmission of sound intensity

The intensity coefficients of reflection and transmission are given by

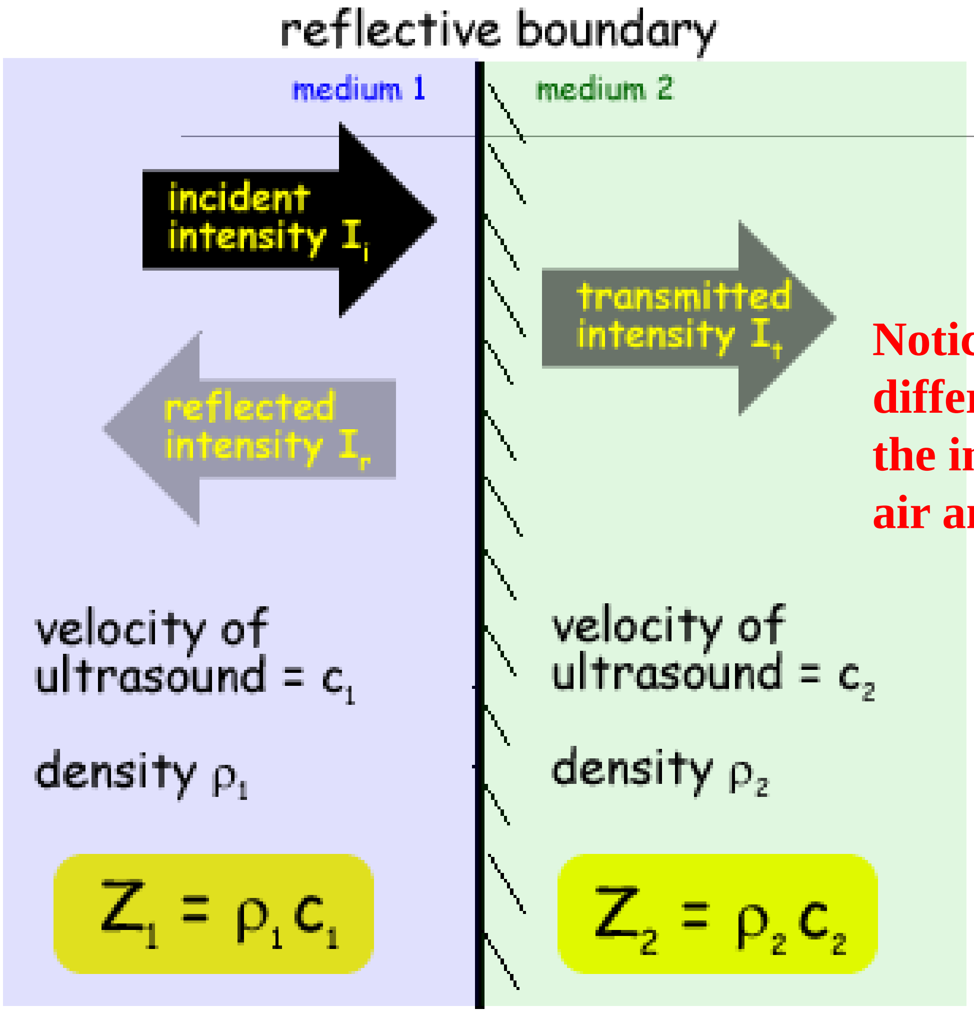
$$\frac{I_r}{I_i} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2 ,$$

$$\frac{I_t}{I_i} = \frac{4Z_1 Z_2}{\left(Z_1 + Z_2 \right)^2}$$

The conservation of energy gives $\frac{I_r}{I_i} + \frac{I_t}{I_i} = 1$

Note : the expressions are applied for normal incidence only.

Acoustic impedance



Notice : a large difference between the impedance of air and skin

Table 4.3 Acoustic Impedance for Specular Reflection by Acoustic Waves Crossing a Material Interface[536,628,629,730,763]

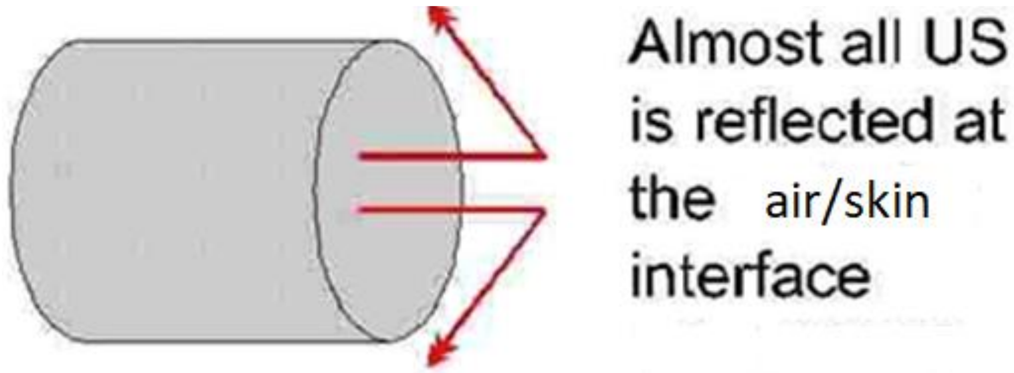
Body Tissue	Impedance (kg/m^2-sec)	Body Tissue	Impedance (kg/m^2-sec)
Air	400	Liver (25°C)	1.65×10^6
Lung	1.80×10^5	Blood	1.65×10^6
Fat	1.39×10^6	Nerve (optic)	1.68×10^6
Aqueous humor	1.51×10^6	Muscle	1.73×10^6
Water	1.52×10^6	Lens of eye	1.84×10^6
Brain (25°C)	1.57×10^6	Nylon	2.9×10^6
Skin	1.6×10^6	Skull bone	7.80×10^6
Soft tissue (avg.)	1.63×10^6	Enamel	1.71×10^7
Kidney	1.63×10^6	Diamond	6.3×10^7

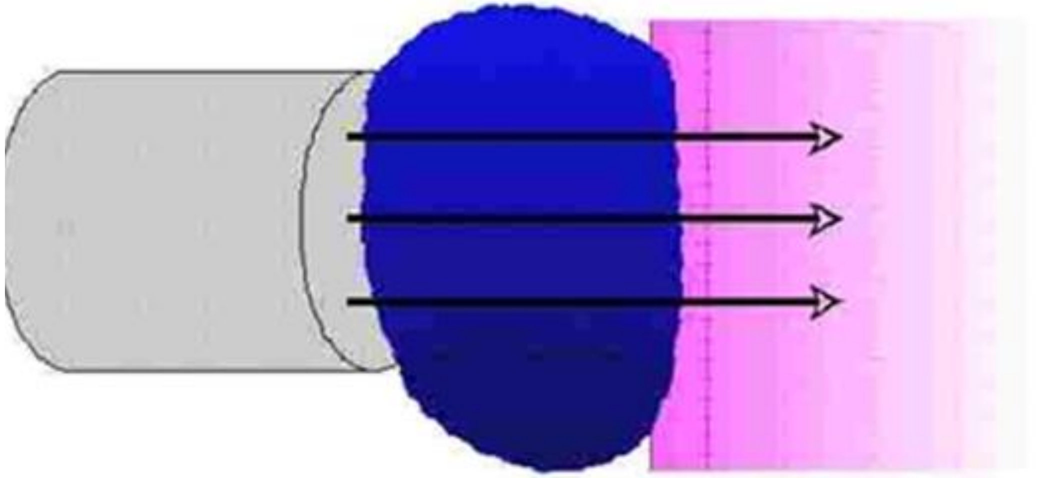
$$\frac{I_r}{I_o} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

http://www.cyberphysics.co.uk/topics/medical/Ultrasound/reflectionUS.html

http://www.nanomedicine.com/NMI/Tables/4.3.jpg

Ultrasound coupling gel

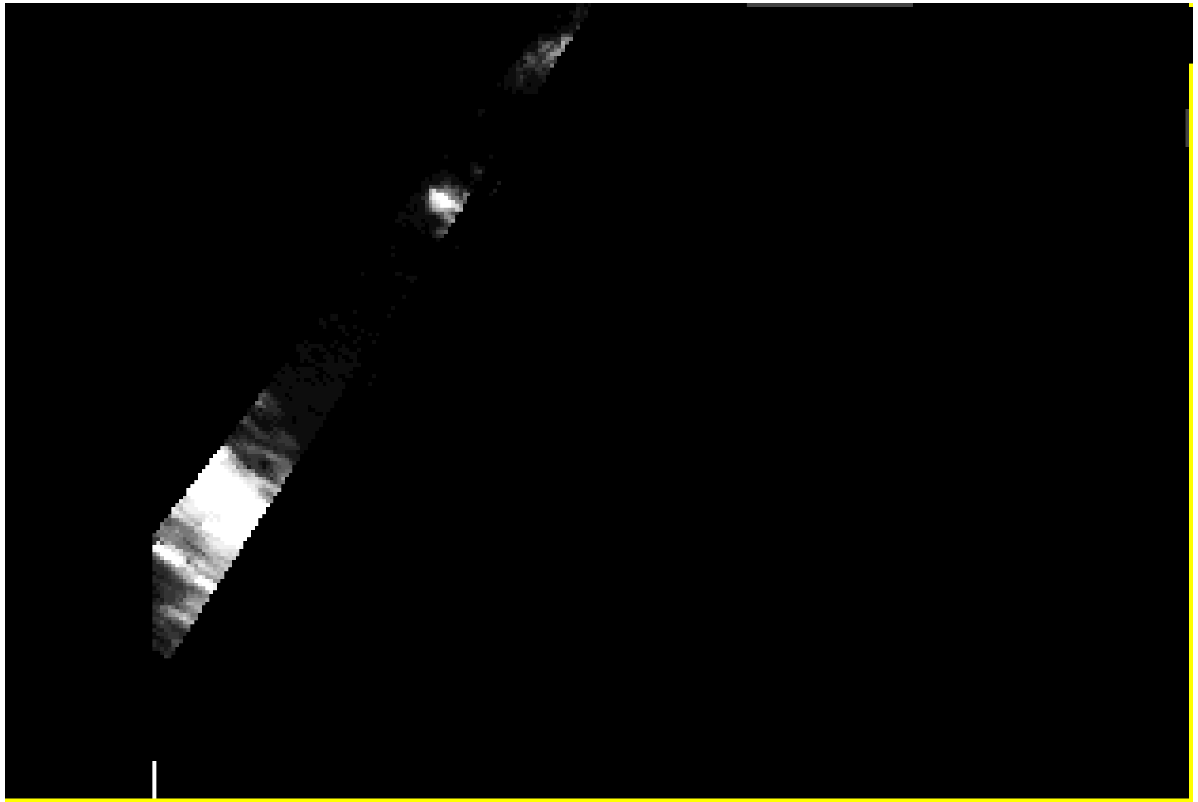


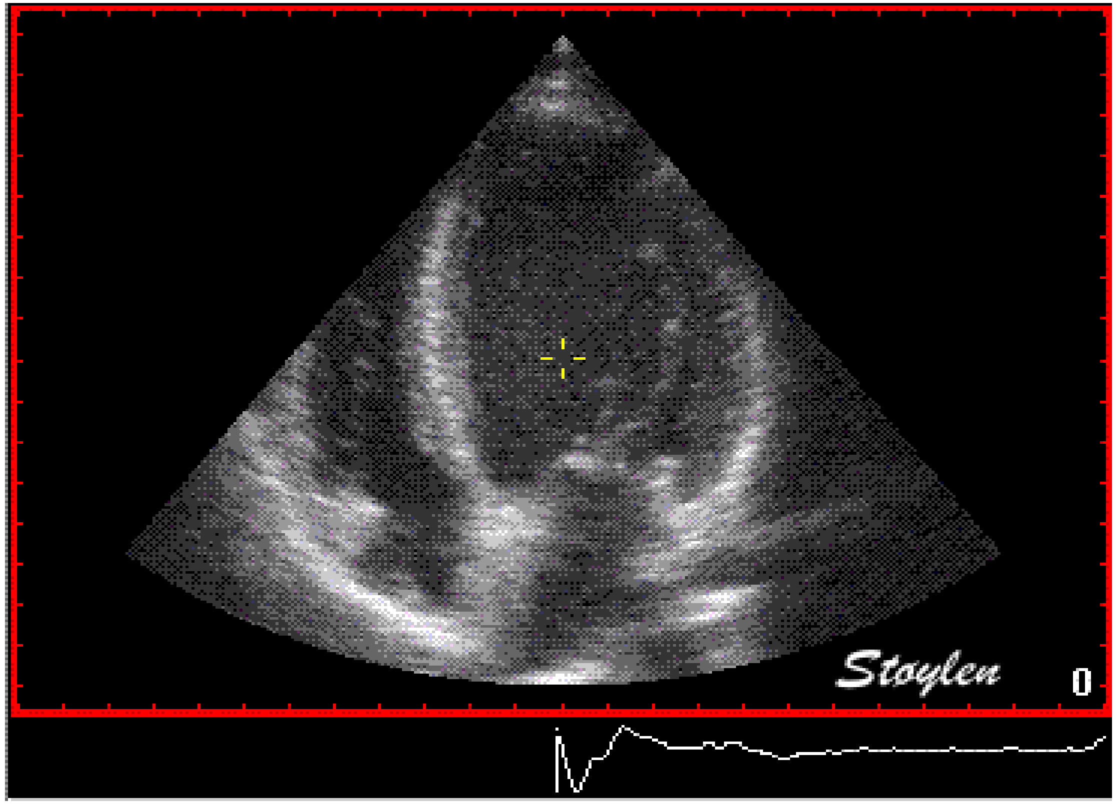


Transmission through coupling medium

- From the table in the previous slide, a large difference between the impedance of air and skin can be seen.
- Without a coupling gel, only a tiny power of ultrasound is sent through the body!
- Recall, $$\left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$
- When Z_1 = 400 kg/m^2-sec and Z_2 = 1.6 x 10^6 kg/m^2-sec
- The reflected intensity is about 99.95%!

A sonogram from ultrasound scan





Example 6 : Reflection and transmission

- Determine the **percentage of energy reflection** when sound waves are normally incident on a plane steel water interface.
- Determine **the percentage of energy transmission**, if the waves are travelling in water and are normally incident on a plane water-ice interface.

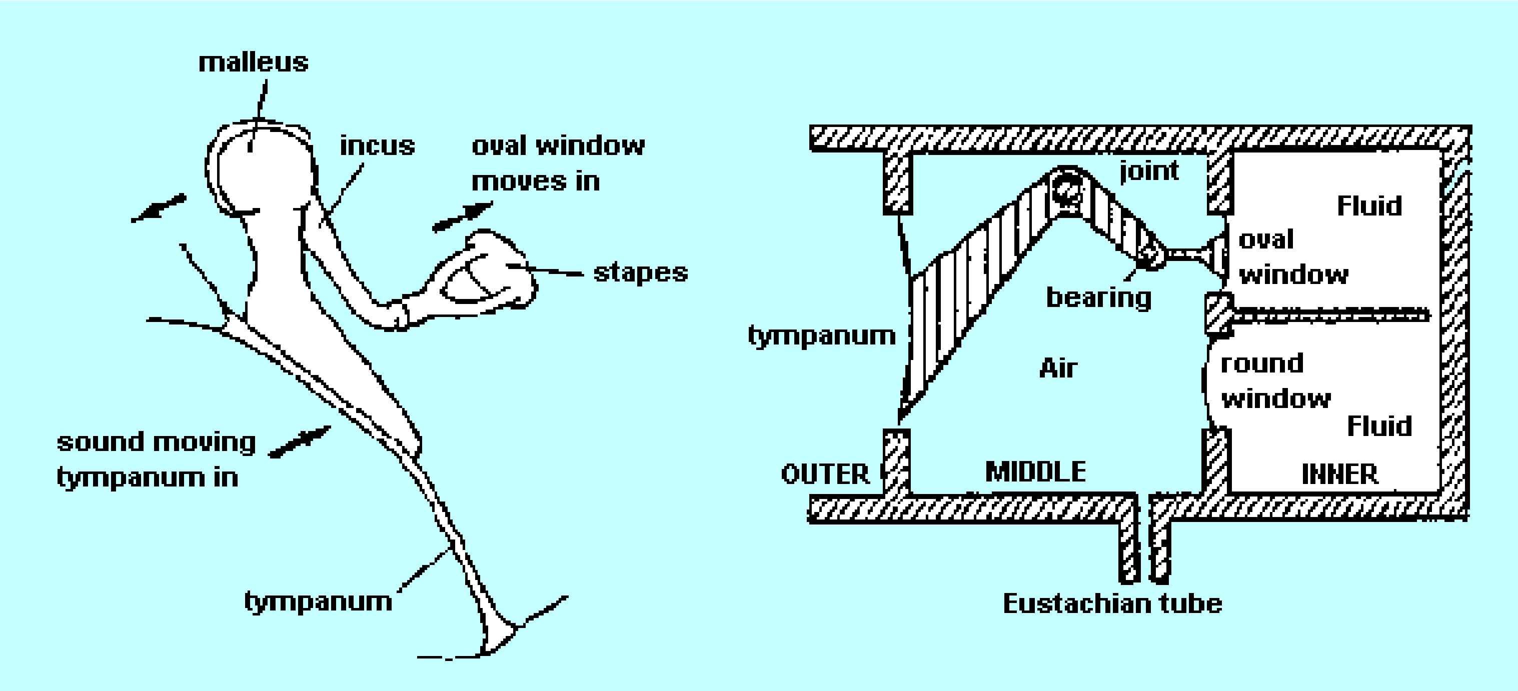
(ρc values in kg m^{-2} s^{-1})

water = 1.43×10^6

ice = 3.49×10^6

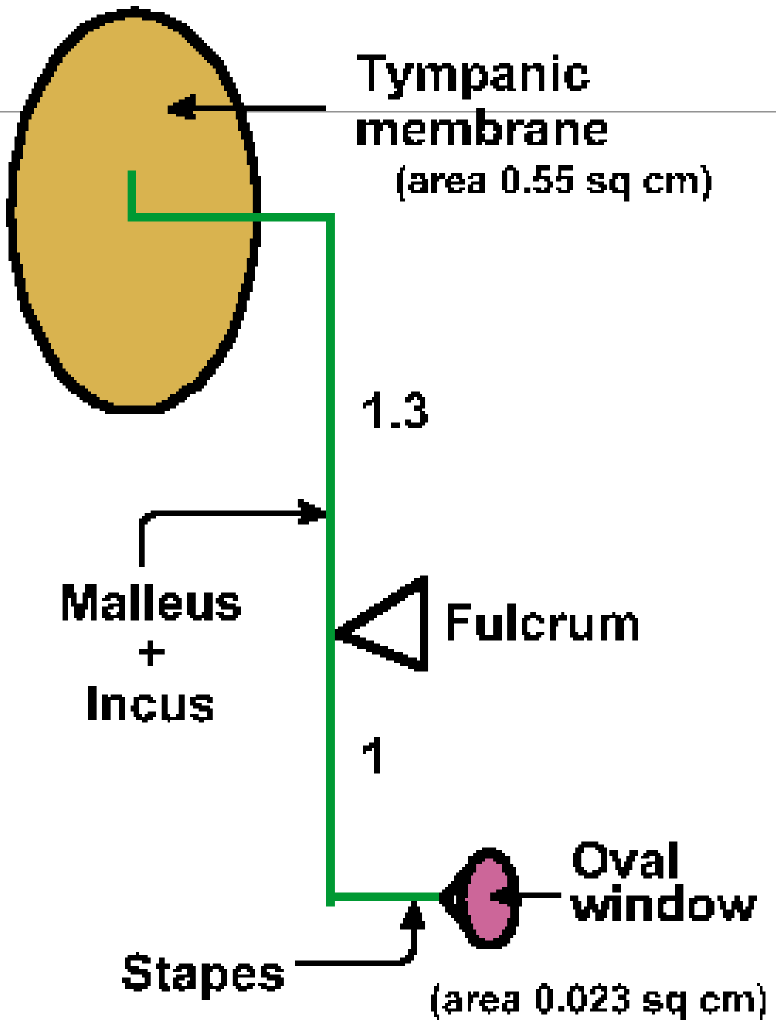
steel = 3.8×10^7

Example 7 : Impedance matching in ear



- Tympanic membrane vibrations are transmitted through middle ear ossicles to the inner ear (cochlea) via the oval window.
- The ossicles provide small amount of amplification, but their main role is **impedance matching.**
- Two main mechanisms involved in the amplification are **area ratio** and **lever action**.

How the middle ear works



$$\frac{P_2}{P_1} = \frac{F_2}{A_2} \div \frac{F_1}{A_1} = \frac{F_2}{F_1} \times \frac{A_1}{A_2}$$

$$\because F_2 > F_1 \text{ and } A_1 > A_2$$

$$\therefore P_2 > P_1$$

P1 : pressure on the tympanic membrane
P2 : pressure on the oval window

Schematic drawing of ossicle system to illustrate the lever arms and the position of the fulcrum. Relative areas of the tympanic membrane and the membrane of the oval window are shown.

http://michaeldmann.net/mann8.html

Longitudinal Waves in a solid

- The velocity of longitudinal waves in a solid depend upon the dimensions of the specimen in which the waves are travelling.

- For a **thin bar** of finite cross section, the wave equation is composed of longitudinal wave from only **longitudinal strain or axial strain** and given as

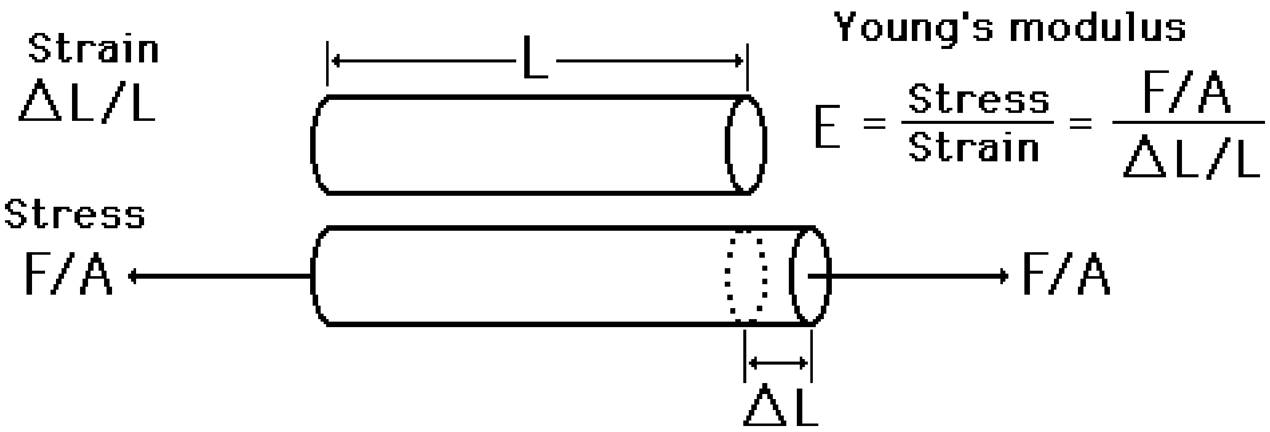
$$\frac{\partial^2 \eta}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \eta}{\partial t^2}, \text{ with } c^2 = \frac{Y}{\rho}$$

Y = Young's modulus, the ratio of the longitudinal stress in the bar to its longitudinal strain.

- **In bulk solids, the wave equations are composed of longitudinal wave and transverse wave due to transverse strain.**

Young’s modulus

Young’s Modulus is a vitally important number that describes how a material behaves under tension.



https://www.quora.com/Does-cold-working-change-Youngs-modulus

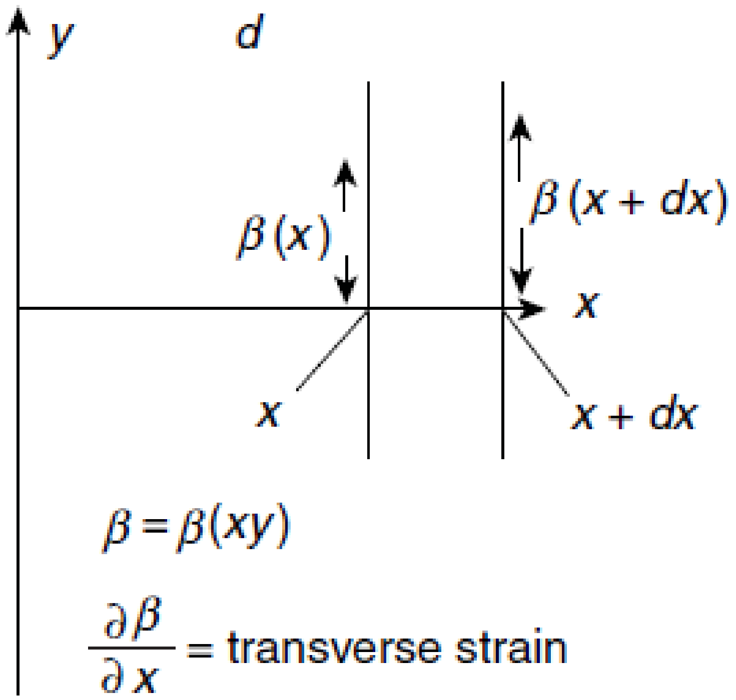
Material	Youngs Modulus /GPa
Mild Steel	210
Copper	120
Bone	18
Plastic	2
Rubber	0.02

http://physicsnet.co.uk/a-level-physics-as-a2/materials/young-modulus/

Transverse wave and transverse velocity in a bulk solid

- Because the **bulk solid** distorts laterally. This gives rise to a transverse wave.
- The transverse shear strain is $\partial\beta/\partial x$ and the transverse shear stress is $\mu(\partial\beta/\partial x)$; μ = shear modulus of rigidity.
- The equation of transverse motion of the thin element dx is given by

summation of transverse shear stress = mass x acceleration



$$T_{x+dx} - T_x = \rho dx \ddot{y}$$

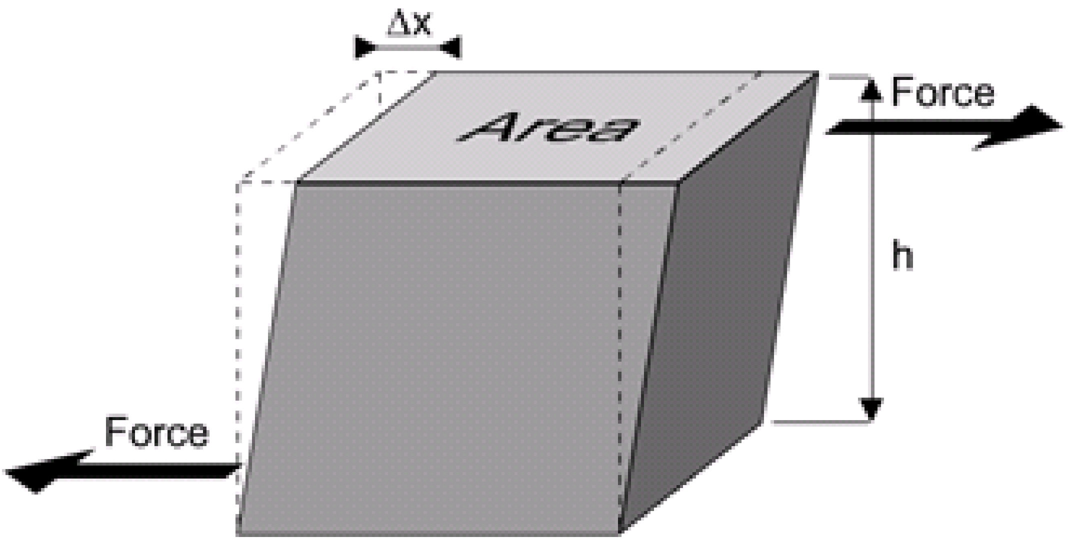
$$\frac{\partial}{\partial x}\left(\mu \frac{\partial \beta}{\partial x}\right) = \rho \ddot{y}$$

$$\frac{\partial^2 \beta}{\partial x^2} = \frac{\rho}{\mu}\frac{\partial^2 \beta}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 \beta}{\partial t^2}$$

β = displacement in y direction and a function of both x and y.

∴ Transverse velocity c = $(\mu/\rho)^{1/2}$

Shear Modulus or modulus of rigidity



- The shearing force do not pass through the same point.
- This result is that the profile of the object becomes distorted.

$$\textbf{Shear modulus } \mu = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{\left(\frac{Force}{Area}\right)}{\left(\frac{\Delta x}{h}\right)}$$

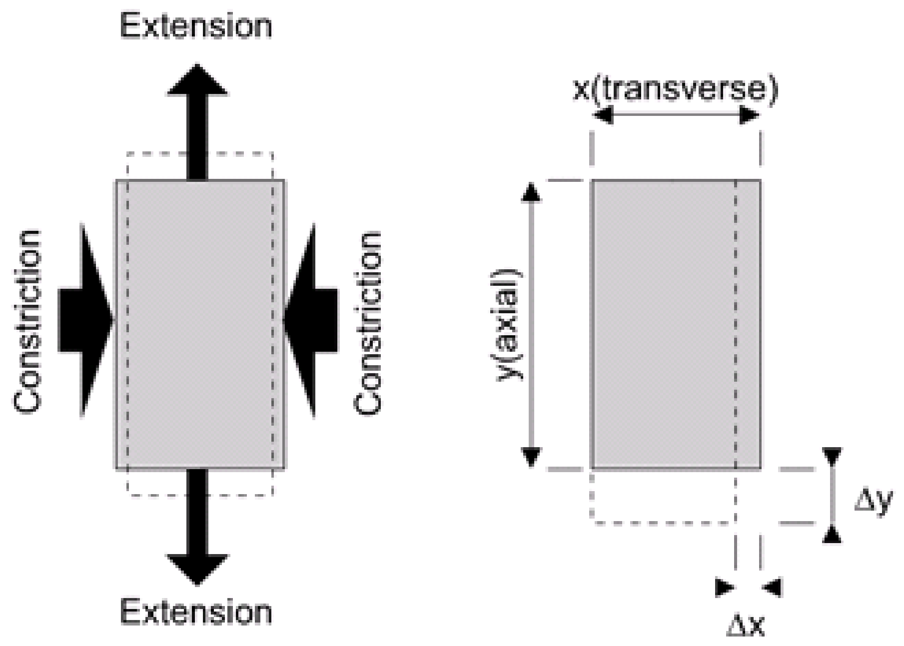
http://www.spaceflight.esa.int/impress/text/education/Mechanical%20Properties/MoreModuli.html

Longitudinal wave velocity in a bulk solid

- The effect of the transverse rigidity is to stiffen the solid and **increase the elastic constant** governing the propagation of longitudinal waves.
- The longitudinal wave velocity becomes $c^2 = \dfrac{\lambda + 2\mu}{\rho}$; where λ is a constant
- Alternatively, $B = \lambda + \dfrac{2}{3}\mu$; B = bulk modulus
- The longitudinal wave velocity for a bulk solid becomes $c_L = \left(\dfrac{B + (4/3)\mu}{\rho} \right)^{1/2}$
- And recall the transverse wave velocity for a bulk solid, $c_T = \left(\dfrac{\mu}{\rho} \right)^{1/2}$
- Relationship between Poisson ratio σ and shear modulus μ

Poisson ratio $\sigma = \dfrac{\lambda}{2(\lambda + \mu)}$

Poisson's ratio



- When a sample, such as a wire, is stretched then it gets a little thinner. The Poisson Ratio puts a number to this effect.
- The axial direction is the direction of the applied force, be that tensional or compressive. The (two) transverse directions are at right angles to this.
- If the sample is stretched then (usually) it will get thinner and so the Poisson Ratio is defined as

$$\text{Poisson Ratio} = -\frac{\text{Transverse Strain}}{\text{Axial Strain}}$$

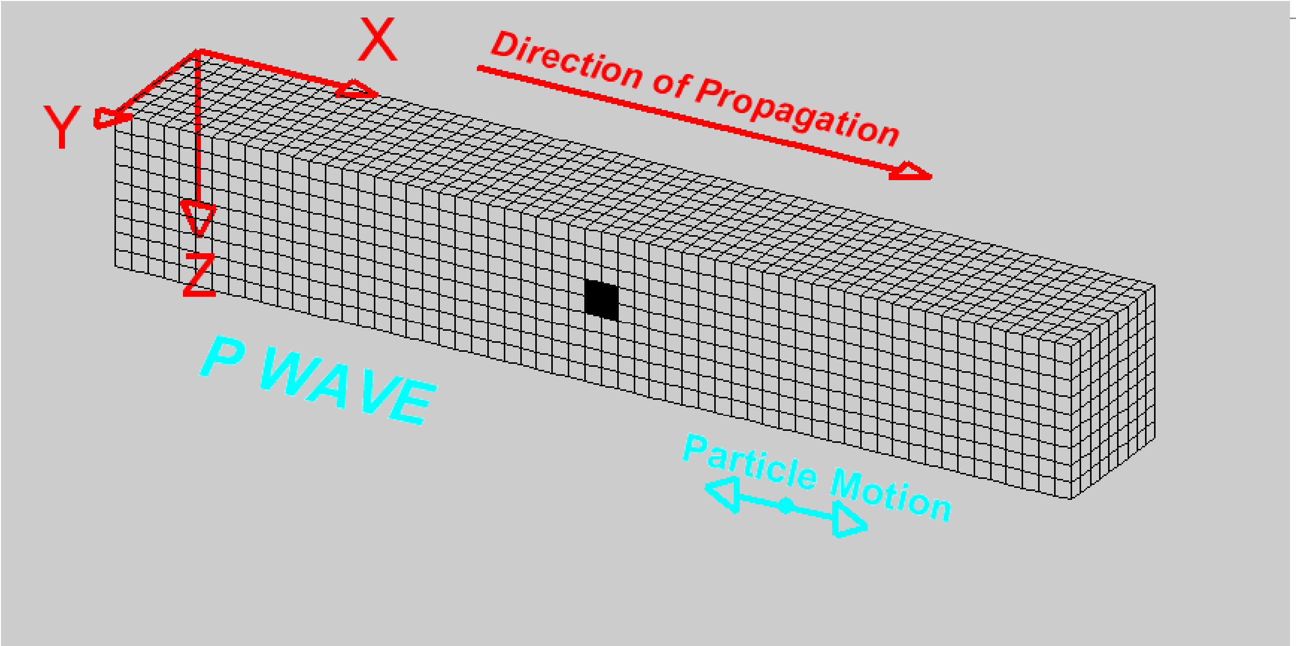
$$= -\frac{\left(\Delta x / x\right)}{\left(\Delta y / y\right)}$$

Example 8 : Poisson ratio for the earth

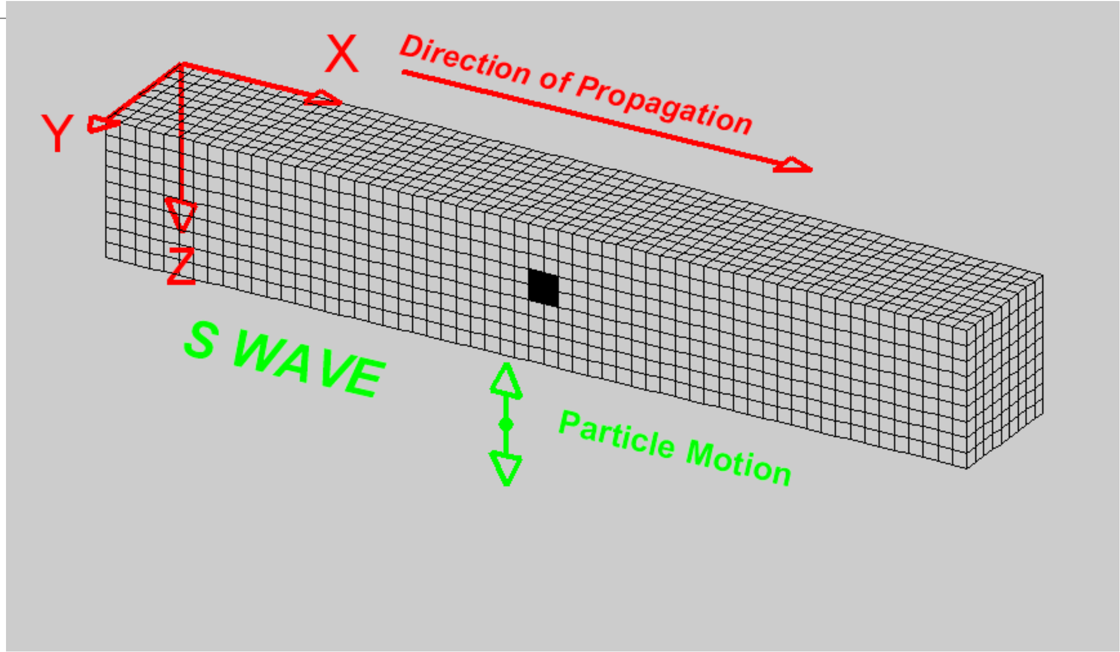
Near the surface of the earth, the longitudinal waves (P wave) have a velocity of 8 km/s and the transverse wave (S wave) travel at 4.45 km/s.

Calculate the Poisson ratio for the earth.

Seismic waves : body waves (P and S waves)



P-waves consists of the transmission of compressions and rarefraction of rock, similar to the propagation of the sound



S-waves, consists of wave propagation of shear, where the particles move in a direction perpendicular to the direction of propagation of the disturbance

https://web.ua.es/en/urs/disclosure/seismic-wave-propagation.html

Homework #6

Problem 6.13

On p. 121 we discussed the problem of matching two strings of impedances Z_1 and Z_3 by the insertion of a quarter wave element of impedance

$$Z_2 = (Z_1 Z_3)^{1/2}$$

Repeat this problem for the acoustic case where the expressions for the string displacements

$$y_i, y_r, y_t$$

now represent the appropriate acoustic pressures p_i, p_r and p_t.

Show that the boundary condition for pressure continuity at $x = 0$ is

$$A_1 + B_1 = A_2 + B_2$$

and that for continuity of particle velocity is

$$Z_2(A_1 - B_1) = Z_1(A_2 - B_2)$$

Similarly, at $x = l$, show that the boundary conditions are

$$A_2 e^{-ik_2 l} + B_2 e^{ik_2 l} = A_3$$

and

$$Z_3(A_2 e^{-ik_2 l} - B_2 e^{ik_2 l}) = Z_2 A_3$$

Hence prove that the coefficient of sound transmission

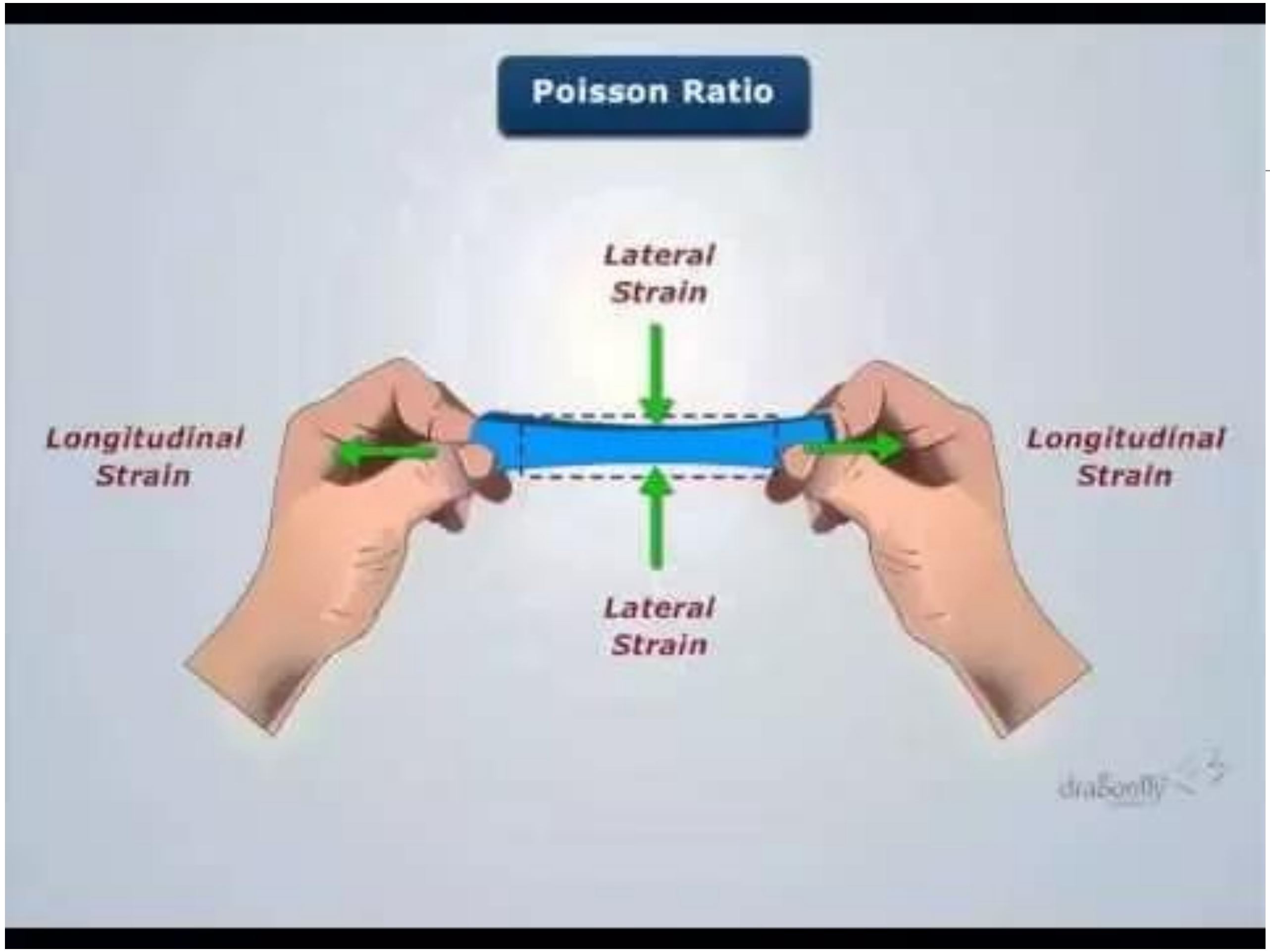
$$\frac{Z_1}{Z_3} \frac{A_3^2}{A_1^2} = 1$$

when

$$Z_2^2 = Z_1 Z_3 \quad \text{and} \quad l = \frac{\lambda_2}{4}$$

(Note that the expressions for both boundary conditions and transmission coefficient differ from those in the case of the string.)

Poisson’s ratio



Poisson’s ratio is a material property describing the lateral strain with respect to the longitudinal stain.

Strain : By how much times something elongated and shortened with respect to original dimension.

Longitudinal direction : Length wise direction, but here its considered the direction of pull.

Lateral direction :Direction perpendicular to longitudinal, here its considered the perpendicular direction with respect to the direction of pull.

Poisson’s effect: Material contracting or expanding in transverse direction when pulled or compressed in axial direction.

Longitudinal Strain : By how much times something elongated and shortened with respect to original dimension in direction of pull(pull is lengthwise).

Lateral/Transverse Strain : By how much times something elongated and shortened with respect to original dimension in perpendicular direction of pull